Book of Abstracts

Workshop
Ideal Fluids and Transport

Institute of Mathematics of Polish Academy of Sciences in Warsaw
Banach Center, Warsaw, 13-15 February 2017

Banach Center
INSTITUTE OF MATHEMATICS PAS

Warsaw Center
of Mathematics
and Computer Science

Schedule

	Monday, February 13	Tuesday, February 14	Wednesday, February 15
8:30–9:00	Registration and openning		
9:00–9:45	YANN BRENIER *Magnetohydrodynamic approach to the curve-shortening flow*	GUI-QIANG CHEN *Supersonic Flow onto Solid Wedges, Multidimensional Shock Waves and Free Boundary Problems*	ATHANASIOS E. TZAVARAS *On the structure of the equations of adiabatic thermoelasticity*
9:45–10:30	JOSÉ L. RODRIGO *On Prandtl-like equation arising from SQG*	STEFANO BIANCHINI *A uniqueness result for the decomposition of vector fields in* $\mathbb{R}^d$	MARIA COLOMBO *Nonlocal-to-local limit of conservation laws*
10:30–11:00	Coffee break	Coffee break	Coffee break
11:00–11:45	GIANLUCA CRIPPA *Exponential self-similar mixing by incompressible flows*	LAURA SPINOLO *Schaeffer's regularity theorem for scalar conservation laws does not extend to systems*	ONDŘEJ KREML *On the Riemann problem for multiD compressible isentropic Euler equations*
11:45–12:30	CHRISTIAN SEIS *Optimal stability estimates for continuity equations*	EDUARD FEIREISL *Solvability of problems involving inviscid fluids*	CHRISTIAN KLINGENBERG *On recent progress in multi-dimensional Euler equations, theory and numerics*
12:30–14:30	Lunch break	Lunch break	
14:30–15:15	CAMILLO DE LELLIS *The Onsager's Theorem*	DENIS SERRE *The subsonic Chaplygin flow: a (very) degenerate elliptic boundary-value problem*	
15:15–16:00	SARA DANERI *The Cauchy problem for dissipative Hölder flows*	MÁRIA LUKÁČOVÁ *Asymptotic preserving schemes for singular limits of compressible flows*	
16:00–16:30	Coffee break	Coffee break	
16:30–17:15	JOSÉ A. CARRILLO *Hydrodynamic Models with Attractive-Repulsive and Alignment Effects*	Poster session	
17:15	THIERRY GALLAY *Axisymmetric vortex rings in ideal and viscous fluids*	Banquet	

A uniqueness result for the decomposition of vector fields in $\mathbb{R}^d$

Stefano Bianchini, Paolo Bonicatto

SISSA, Trieste

Abstract

Given a vector field $\rho(1, \mathbf{b}) \in L^1_{\mathrm{loc}}(\mathbf{R}^{d+1}, \mathbb{R}^{d+1})$ such that $\mathrm{div}_{t,x}(\rho(1, \mathbf{b}))$ is a measure, we consider the problem of uniqueness of the representation η of $\rho(1, \mathbf{b})\mathcal{L}^{d+1}$ as a superposition of characteristics $\gamma : (t^-_\gamma, t^+_\gamma) \to \mathbb{R}^d$, $\dot\gamma = \mathbf{b}(t, \gamma(t))$. We give conditions in terms of a local structure of the representation η on suitable sets in order to prove that there is a partition of $\mathbb{R}^{d+1}$ into disjoint trajectories $\mathrm{p_a}$, $\mathrm{a} \in \mathrm{A}$, such that the PDE

$$\mathrm{div}_{t,x}\big(u\rho(1, \mathbf{b})\big) \in \mathcal{M}(\mathbb{R}^{d+1}), \qquad u \in L^\infty(\mathbb{R}^{d+1}),$$

can be disintegrated into an ODE along $\mathrm{p_a}$ with measure r. h. s. The decomposition $\mathrm{p_a}$ is essentially unique. We finally show that $\mathbf{b} \in L^1_t(\mathrm{BV}_x)_{\mathrm{loc}}$ satisfies this local structural assumption, yielding in particular the renormalization properties for nearly incompressible BV vector fields.

Magnetohydrodynamic approach to the curve-shortening flow

Yann Brenier

CNRS, École Polytechnique, Palaiseau

Abstract

The curve-shortening flow is the simplest example of mean-curvature flow with co-dimension larger than one. We show how this problem can be translated in magnetohydrodynamic terms. A relative entropy method is used to get global "dissipative solutions" (in the sense of P.-L. Lions) enjoying a "strong-weak" uniqueness principle.

Hydrodynamic Models with Attractive-Repulsive and Alignment Effects

José A. Carrillo

Imperial College London

Abstract

I will discuss several recent results regarding qualitative properties of hydrodynamic models with attractive-repulsive interactions. These models appear as natural monokinetic closures or hydrodynamic solutions of kinetic models for collective behavior. We will discuss critical thresholds and long time asymptotics for 1D models with alignment with/without interaction forces. In the pure alignment case and in the pure Euler-Poisson with confinement we will give sharp result distinguishing global existence and finite time blow-up with a complete understanding of the long time asymptotics. This is a summary of results based on works in collaboration with Y.-P. Choi, E. Tadmor, and C. Tan and with Y.-P. Choi and E. Zatorska.

Supersonic Flow onto Solid Wedges, Multidimensional Shock Waves and Free Boundary Problems

Gui-Qiang Chen

University of Oxford

Abstract

When an upstream steady uniform supersonic flow, governed by the Euler equations, impinges onto a symmetric straight-sided wedge, there are two possible steady oblique shock configurations if the wedge angle is less than the detachment angle – the steady weak shock with supersonic or subsonic downstream flow (determined by the wedge angle that is less or larger than the sonic angle) and the steady strong shock with subsonic downstream flow, both of which satisfy the entropy conditions. The fundamental issue – whether one or both of the steady weak and strong shocks are physically admissible solutions – has been vigorously debated over the past eight decades. In this talk, we discuss some recent developments on the stability analysis of the steady shock solutions in both the steady and dynamic regimes. The corresponding stability problems can be formulated as free boundary problems for nonlinear partial differential equations of mixed elliptic-hyperbolic type, whose solutions are fundamental for the multidimensional theory of hyperbolic conservation laws. Some further developments, open problems, and mathematical challenges in this direction are also addressed.

Nonlocal-to-local limit of conservation laws

Maria Colombo

University of Zurich, ETH Zurich

Abstract

We consider a nonlocal approximation of the Burgers' equation obtained by convolving the velocity field with a smooth, compactly supported convolution kernel ρ_ε in some of its occurrencies

$$\partial_t u^\varepsilon + \partial_x((u^\varepsilon * \rho_\varepsilon)u^\varepsilon) = 0.$$

As ε goes to 0, Zumbrun proved the convergence of u^ε to a solution of the Burgers' equation holds provided that the convolution kernel is even and the limit is sufficiently smooth.

Motivated by numerical simulations, in particular by Amorim, Colombo and Teixeira, we investigate the convergence when the limit solution is nonsmooth, providing several counterexamples and some positive results for the corresponding viscous problem.

Exponential self-similar mixing by incompressible flows

Gianluca Crippa

University of Basel

Abstract

I will address the problem of the optimal mixing of a passive scalar under the action of an incompressible flow in two space dimensions. The scalar solves the continuity equation with a divergence-free velocity field which satisfies a bound in the Sobolev space $W^{s,p}$, where $s \geq 0$ and $1 \leq p \leq \infty$. The mixing properties are given in terms of a characteristic length scale, called the mixing scale. We consider two notions of mixing scale, one functional, expressed in terms of the homogeneous Sobolev norm $\dot{H}^{-1}$, the other geometric, related to rearrangements of sets. We study rates of decay in time of both scales under self-similar mixing. For the case $s = 1$ and $1 \leq p \leq \infty$ (including the Lipschitz case, and the case of physical interest of enstrophy-constrained flows), we present examples of velocity fields and initial configurations for the scalar that saturate the exponential lower bound established in previous works for the decay in time of both scales. We also obtain several consequences for the geometry of regular Lagrangian flows associated to Sobolev velocity fields and for the loss of regularity for continuity equations with non-Lipschitz velocity field. The talk will be based on joint works with G. Alberti (University of Pisa) and A. L. Mazzucato (Penn State).

The Cauchy problem for dissipative Hölder Euler flows

Sara Daneri

University of Erlangen-Nüremberg

Abstract

We address the Cauchy problem for the incompressible Euler equations in a periodic setting. It was recently proven [5] that there exist infinitely many non-conservative solutions for the $3d$-Euler equations in the Hölder class $C^{0,\alpha}$, for every $\alpha < 1/3$. In [1] the authors refined the result, showing that one can get such solutions even imposing that the energy is dissipated. Our results aims at showing that, below the Onsager's critical regularity of Hölder $1/5$ in space, for an L^2-dense set of initial data, the Euler equations are ill-posed. In particular, dissipation of energy does not help to restore uniqueness, as instead happens in the smooth case. Our results are prior to the latest results of [5] and [1] and introduce a new set of stationary solutions of the Euler equations which have been then used to prove the full Onsager's conjecture.

References

[1] Buckmaster, T., De Lellis, C., Székelyhidi, Jr., L. and Vicol, V. *Onsager's conjecture for admissible weak solutions*, arXiv: 1701.08678

[2] Buckmaster, T., De Lellis, C., Isett, P. and Székelyhidi, Jr., L. *Anomalous dissipation for* 1/5-*Hölder Euler flows* Ann. of Math. (2) 182 (2015), no. 1, 127–172.

[3] Daneri, S. *Cauchy problem for dissipative Hölder solutions to the incompressible Euler equations* Comm. Math. Phys. 329 (2014), no. 2, 745–786.

[4] Daneri, S. and Székelyhidi, Jr., L. *Non-uniqueness and h-principle for Hölder-continuous weak solutions of the Euler equations* Arch. Rat. Mech. Anal. (2017) DOI:10.1007/s00205-017-1081-8.

[5] Isett, P. *A proof of Onsager's conjecture* arXiv:1608.08301.

The Onsager's Theorem

Camillo De Lellis

University of Zurich

Abstract

In 1949 the famous physicist Lars Onsager made a quite striking statement about solutions of the incompressible Euler equations: if they are Hoelder continuous for an exponent larger than $\frac{1}{3}$, then they preserve the kinetic energy, whereas for exponents smaller than $\frac{1}{3}$ there are solutions which do not preserve the energy. The first part of the statement has been rigorously proved by Constantin, E and Titi in the nineties. In a series of works László Székelyhidi and myself have introduced ideas from differential geometry and differential inclusions to construct nonconservative solutions and started a program to attack the other portion of the conjecture. After a series of partial results, due to a few authors, Phil Isett has recently fully resolved the problem. In this talk I will try to describe as many ideas as possible and will therefore touch upon the works of several mathematicians, including László, Phil, Tristan Buckmaster, Sergio Conti, Sara Daneri, Vlad Vicol and myself.

Solvability of problems involving inviscid fluids

Eduard Feireisl

Czech Academy of Sciences

Abstract

We discuss solvability/well posedness of ceratin problems involving inviscid fluids, notably the compressible Euler system and its variants. We review certain results obtained via convex integration, introduce the concept of measure valued solutions, and discuss suitable admissibility criteria.

Axisymmetric vortex rings in ideal and viscous fluids

Thierry Gallay

University of Grenoble

Abstract

A vortex ring is a flow with the property that the vorticity distribution is essentially concentrated in a solid torus. For the Euler equations with rotational symmetry, uniformly translating vortex rings can be constructed by variational techniques or by a fixed point argument. In the viscous case, it is natural to study the Cauchy problem for the axisymmetric Navier-Stokes equations, assuming that the initial vorticity is a circular vortex filament. We prove that the corresponding solution is unique and evolves into a viscous vortex ring which can be accurately described, at least for small times. This a joint work with Vladimír Šverák (Minneapolis).

On recent progress in multi-dimensional Euler equations, theory and numerics

Christian Klingenberg

University of Würzburg

Abstract

We will present results concerning the two-dimensional compressible inviscid Euler equations. We begin by considering 1-d Riemann initial data in the x-direction that is extended into 2-d by making it independent of the y-component. It shown that for all standard solutions of the 1-d Riemann solution (extended into 2-d) give rise to so-called “wild” solutions, except if the standard solution is smooth. We continue to look at numerical schemes for the 2-d Euer equations. We demand such schemes to be stationary preserving. For the case of linearized Euler, such scheme can be characterized and shown to have to be asymptotic preserving for low Mach numbers. - This is joint work with Simon Markfelder, Wasilij Barsukow and Phil Roe.

On the Riemann problem for multiD compressible isentropic Euler equations

Ondřej Kreml

Czech Academy of Sciences

Abstract

We consider the Riemann problem for the two-dimensional compressible isentropic Euler system, i.e. we solve the equations on the whole 2D space and the initial data consists of two pairs of constants, each on one of the halfplanes. This classical problem possesses a one-dimensional self-similar admissible solution which is unique in the BV class. We study uniqueness of admissible solutions in the broader class of essentially bounded functions. We summarize the up-to-date results concerning uniqueness and nonuniqueness of admissible weak solutions in certain cases of initial data. The nonuniqueness results are based on the theory of De Lellis and Szekelyhidi for incompressible Euler equations, whereas the uniqueness results are consequences of the relative energy inequality developed recently by Feireisl, Novotny and coauthors. All of the presented results hold for any space dimension larger than one.

On Prandtl-like equation arising from SQG

José L. Rodrigo

University of Warwick

Abstract

In this talk I will describe joint work with C. Fefferman on a Prandtl-like equation arising when considering the evolution of families of almost-sharp fronts for the surface quasi-geostrophic equation. We will introduce all these objects and show how to solve the equation (under appropriate monotonicity conditions - like in Prandtl) in the smooth case.

Asymptotic preserving schemes for singular limits of compressible flows

Mária Lukáčová

University of Mainz

Abstract

In this talk we will present our recent results on singular limits of compressible flows as the Mach number approaches zero. Such weakly compressible flows have complex multiscale behaviour since flow velocity is much smaller than the sound speed and the speed of gravity waves.

In order to efficiently resolve low Mach number flows it is suitable to split the whole system into the linear operator describing the evolution of the linearized acoustic and gravity waves and the nonlinear operator for advective transport. Due to the multiscale behaviour the linear operator is stiff, which suggests to use an implicit time approximation. On the other hand the nonlinear transport is non-stiff and will be approximated by an explicit scheme. Such a combination yields the IMEX (implicit-explicit) finite volume (FV) scheme and allows efficient approximation of the compressible Euler equations. We show theoretically as well as by means of numerical experiments that the IMEX FV scheme is asymptotic preserving and yields a consistent approximation of the incompressible limiting equations.

Moreover, in the case when the viscous effects are included we are able to prove that the error estimates for the implicit mixed finite element-finite volume scheme are asymptotic preserving as well. The proof is based on the use of the relative entropy. We will also discuss the case when the adiabatic exponent is less than 3/2 and show that the numerical solutions converge to the measure valued solutions.

The present research has been obtained in the collaboration with E. Feireisl, A. Novotny, S. Necasova, B. She, G. Bispen and L. Yelash. It has been supported by the DFG under TRR 146 "Multiscale Simulation Methods for Soft Matter" and TRR 165 "Waves to Weather".

Optimal stability estimates for continuity equations

Christian Seis

Hausdorff Center for Mathematics, Bonn

Abstract

In their ground breaking '89 paper, R. J. DiPerna and P.-L. Lions study uniqueness and stability of solutions to linear continuity equations with Sobolev coefficients and rough data. What is not contained in their "theory of renormalized solutions" are quantitative stability estimates which allow to control the distance of two solutions if coefficients or data are varied. Such estimates, however, are indispensable in the analysis of a number of applications in fluid dynamics.

In this talk, I will present a new quantitative approach to continuity equations. This approach is based on stability estimates that are formulated in terms of Kantorovich–Rubinstein distances with logarithmic costs. I will show how the new estimates can be applied to obtain optimal bounds on the order of convergence of the numerical upwind scheme or on the rate of mixing by stirring of fluids. This is partially joint work with A. Schlichting.

The subsonic Chaplygin flow: a (very) degenerate elliptic boundary-value problem

Denis Serre

École normale supérieure de Lyon

Abstract

The two-dimensional Riemann problem for a Chaplygin gas can be solved at bare hands within the supersonic domain. In particular, we do know the exact shape of its complement, the subsonic domain. Within the latter, the velocity potential obeys a second order elliptic equation with the homogeneous Dirichlet data. This highly degenerate BVP turns out to govern the complete minimal surfaces in the constant negative curvature Riemannian space. An obvious necessary condition for the existence is the convexity of the domain Ω. We prove that this is also a sufficient one. The amazing proof of the key estimate, that the solution is locally uniformily Lipschitz, involves the Hilbert distance in Ω. Incidentally, our work gives a new, more natural, proof that the latter is a distance.

Schaeffer's regularity theorem for scalar conservation laws does not extend to systems

Laura Spinolo

IMATI, Pavia

Abstract

In 1973 Schaeffer established a regularity result that applies to scalar conservation laws with convex fluxes and can be loosely speaking formulated as follows: for a generic smooth initial datum, the admissible solution is smooth outside a locally finite number of curves in the (t, x) plane. Here the term "generic? should be interpreted in a suitable technical sense, related to the Baire Category Theorem. My talk will aim at discussing an explicit counter-example that shows that Schaeffer?s theorem does not extend to systems of conservation laws. The talk will be based on joint works with Laura Caravenna.

On the structure of the equations of adiabatic thermoelasticity

Athanasios E. Tzavaras

KAUST, Saudi Arabia

Abstract

In this talk we review a relative entropy framework for hyperbolic-parabolic systems of conservation laws. We then apply this result together with some transport identities for the null-Lagrangeans to study the structure of the system of adiabatic thermoelasticity. The equations of adiabatic thermoelasticity is a paradigm of a system of conservation laws where the lack of uniform convexity of the entropy poses challenges in the mathematical theory. Nevertheless, the existence of certain nonlinear transport equations for null-Lagrangeans reinforces the efficacy of the entropy as a stabilizing factor and recovers the strength associated wth uniformly convex entropies in hyperbolic systems. The system can be embedded into a larger symmetric hyperbolic system and visualized as constrained evolution which leads to a weak-strong uniqueness theorem and convergence results for approximate solutions in the smooth regime.

Poster session

- Rafael Granero Belinchón (University of Lyon) - *On Rayleigh-Taylor instability and mixing*,
- Simon Markfelder (University of Würzburg) - *On uniqueness of entropy solutions to the isentropic compressible Euler equations in 2D*,
- Martin Michálek (Czech Academy of Sciences) - *Existence of global weak solutions for primitive equations.*