Workshop Nonstandard Growth Analysis and its Applications 2017

Book of abstracts

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Alexandre Almeida (University of Aveiro)
Approximation in Morrey spaces

We introduce a new subspace of Morrey spaces where the approximation by nice functions is possible in Morrey norm. In particular, we obtain an explicit description of the closure of the set of all infinitely differentiable compactly supported functions in Morrey spaces.

This is based on a joint work with S. Samko.

Sami Aouaoui (ISMAI Kairouan, University of Kairouan, Tunisia)
Multiple solutions to some degenerate quasilinear equation with variable exponents via perturbation method

In this paper, we prove the existence of multiple solutions to the following degenerate quasilinear elliptic equation

$$(P_\lambda)\ -\text{div}\left(a(x,u)\left|\nabla u\right|^{p(x)-2}\nabla u\right)+\frac{a_s'(x,u)}{p(x)}\left|\nabla u\right|^{p(x)}+\left|u\right|^{p(x)-2}u=\lambda g(x)\left|u\right|^{q(x)-2}u$$

in $\mathbb{R}^N$, $N \geq 3$, where p is some Lipschitz continuous function such that $1 < p^- < p^+ < N,\ q \in \mathcal{C}(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$ is such that $1 < q^- \leq q(x) < p^*(x),\ \forall\, x \in \mathbb{R}^N$, and $q^+ < 1 + (p^-)^* - p^-,\ \lambda$ is a positive parameter and $a \in \mathcal{C}(\mathbb{R}^N \times \mathbb{R},\ ]0,+\infty[)$ is of class C^1 with respect to $s \in \mathbb{R}$ uniformly for $x \in \mathbb{R}^N$. Moreover, we assume that there exist three positive constants $a_0,\ \alpha$ and C_0 such that

$$a_s'(x,s)s \geq 0,\ a_0 \leq a(x,s) \leq C_0 a(x,|s|^\alpha),\ \forall\,(x,s) \in \mathbb{R}^N \times \mathbb{R}.$$

We also assume that $g \in L^{\sigma(x)}(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N),\ \sigma(x) = \frac{p^*(x)}{p^*(x)-q(x)}$ is such that $g(x) > 0$, $\forall\, x \in \mathbb{R}^N$.

The solutions are distinguished by their signal, i.e we prove the existence of one positive, one negative and one sign-changing solution to (P_λ). Our main idea is to make use of the perturbation method together with invariant sets of descending flow. The fact that the exponents are not constant plays an important role.

Miroslav Bulíček (Charles University)
On nonlinear elliptic equations beyond the natural duality pairing

We establish existence, uniqueness and optimal regularity results for very weak solutions to certain nonlinear elliptic boundary value problems. We provide a unified approach that leads qualitatively to the same theory as that one available for linear elliptic problems with continuous coefficients, e.g. the Poisson equation. The result is based on several novel tools that are of independent interest: local and global estimates for (non)linear elliptic systems in weighted Lebesgue spaces with Muckenhoupt weights, a generalization of the celebrated divcurl lemma for identification of a weak limit in border line spaces and the introduction of a Lipschitz approximation that is stable in weighted Sobolev spaces.

Lars Diening (University of Osnabrück)
Nonlinear Calderon-Zygmund theory

For many linear partial differential equations there is a correlation between the data and the solution in terms of a singular integral operator. Due to this fact the data and the solution are in the same function space. For example the standard Laplace equation with L^p data has solutions with second gradients in L^p. For non-linear partial differential equations this approach is not possible. However, starting with the result of Iwaniec, many similar quantative statements have been derived for the p-Laplace equation/system. In this talk I present a novel pointwise estimate of the gradients of the solution in terms of maximal functions. Many known estimates as well as new endpoint estimates follow from this.

This is a joint work with Dominic Breit, Andrea Chiani, Tuomo Kuusi and Sebastian Schwarzacher.

Sylwia Dudek (Cracow University of Technology)
Liouville-type result for elliptic problems in variable exponent spaces

We will consider nonexistence of nonnegative solutions to a partial differential inequality involving the $p(x)$–Laplacian of the form

$$-\Delta_{p(x)} u \geq \Phi(x, u(x), \nabla u(x))$$

in $\mathbb{R}^n$, where $\Phi(x, u, \nabla u)$ is a locally integrable Carathéodory's function. We assume that $\Phi(x, u, \nabla u) \geq 0$ or compatible with p and u. Growth conditions on u and p lead to Liouville–type results for u.

Moreover, we present the general Liouville–type theorem for problems of $p(x)$–type growth.

[1] S. Dudek, I. Skrzypczak, *Liouville theorems for elliptic problems in variable exponent spaces*, Comm. Pure Appl. Anal. **16** (2) (2017), 513–532.

[2] S. Dudek, *The Liouville–type theorem for problems with nonstandard growth derived by Caccioppoli–type estimate*, under review.

Michał Gaczkowski (Polish Academy of Sciences)
Compact embeddings of variable Sobolev spaces on complete Riemannian manifolds

One can see that Riemannian manifold is a good setting for defining variable exponent Sobolev spaces. After introducing such space we start to study Sobolev embedding. For non compact manifolds they not hold. Inspired by result from Euclidean setting and space of radially symmetrical functions we seek for the other space, where compact embedding are valid. This leads us to variable Sobolev spaces invariant under action of the group. We will present assumptions under which there are compact embeddings. As an application we will study the PDE problem

$$-\Delta_{q(x)}u(x) + |u(x)|^{q(x)-2}u(x) = f(x, u(x)).$$

The talk is based on results obtained together with P. Górka and D. Pons.

[1] M. Gaczkowski, P. Górka, D. Pons, *Sobolev spaces with variable exponents on complete manifolds*, J. Func. Anal., **270** (2016), 1379–1415.

Marek Galewski (Lodz University of Technology)
Multiple solutions to differential inclusions with the $p\,(x)$–Laplacian

Using the Fenchel-Young Duality and Mountain Pass Geometry we derive a multiple critical point theorem for non-smooth functionals. We complete the theoretical approach with the use of the Ekeland Variational Principle. The applications to differential inclusions with the $p\,(x)$–Laplacian and Dirichlet boundary conditions are given.

Paweł Goldstein (University of Warsaw)
Degree theory in Orlicz-Sobolev spaces

One of the crucial topological invariants to study continuous mappings between n-dimensional manifolds is the topological degree. The degree, which can be expressed as the integral of the Jacobian determinant, is also well defined in the Sobolev space $W^{1,n}$, and it is continuous there.

Orlicz-Sobolev mappings slightly larger than $W^{1,n}$ are of particular interest for geometric function theory, in particular for quasiconformal and quasiregular mappings. There appears a natural question: is the degree still well defined (ad if so, how to define it?) and continuous in these spaces? The answer surprizingly depends deeply on the topology of the target manifold.

My talk will be based first on results of Hajłasz, Iwaniec, Malý and Onninen and then on my joint results with Piotr Hajlasz.

Hassane Hjiaj (Université Sidi Mohamed Ben Abdellah)
Existence of entropy solutions
for some anisotropic quasilinear elliptic unilateral problems

In this work, we consider the following quasilinear elliptic unilateral equations of the type

$$-\sum_{i=1}^{N} \frac{\partial}{\partial x_i} a_i(x, u, \nabla u) = \mu - \text{ div } \phi(u) \quad \text{ in } \Omega,$$

in the anisotropic Sobolev space, we prove the existence of entropy solutions for our unilateral problem, where $\mu = f - \text{div } F$ belongs to $L^1(\Omega) + W^{-1,\vec{p'}}(\Omega)$ and $\phi(\cdot) \in C^0(I\!R, I\!R^N)$.

Peter Hästö (Universities of Oulu and Turku)
Regularity theory for Musielak–Orlicz growth

In this talk I present our recent results on regularity of minimizers in the case of generalized Orlicz growth without structure conditions. We obtain Hölder continuity of the minimizers as well as boundary behavior in Lipschitz domains. This is joint work with Petteri Harjulehto and Olli Toivanen.

Piotr Kalita (Jagiellonian University)
Convergence of attractors
for a nonautonomous singularly perturbed semilinear parabolic equation

We consider the initial and boundary value problem governed by the equation

$$u_t - \Delta u = f_0(u)$$

on a bounded domain $\Omega \subset \mathbb{R}^3$ with the homogeneous Dirchlet conditions and cubic nonlinearity f_0. We compare the global attractor of the semiflow governed by the above equation with uniform, pullback, and cocycle attractors of the process governed by its nonautonomous singular perturbation $\epsilon u_{tt} + u_t - \Delta u = f_\epsilon(t, u)$. We prove that all three types of nonautonomous attractors converge both upper- and lower- semicontinuously to the global attractor for the unperturbed problem as $\epsilon \to 0$.

Agnieszka Kałamajska (University of Warsaw)
Trace and extension thereorem in the weighted Orlicz setting

Our goal is to discuss trace embedding and trace extension theorems between weighted Orlicz-Sobolev spaces of functions defined on domain in $\mathbf{R}^n$ and weighted Orlicz-Slobodetski space of functions defined on the boundary of the domain. We will be particularly interested in the class of weights which have the form $\rho = \tau\left(\mathrm{dist}(x, \partial\Omega)\right)$ for weighted Orlicz-Sobolev spaces and weights $\omega(x,y) = |x-y|^{n-2}\tau(|x-y|)$, for weighted Orlicz-Slobodetski spaces, within certain class of functions $\tau(\cdot)$. This gives the new tool to deal with boundary value problems like:

$$\begin{cases} -\mathrm{div}\left(\rho(x)B(\nabla u(x))\right) = f & \text{in} \quad \Omega \\ \qquad u = g & \text{in} \quad \partial\Omega. \end{cases} \tag{1}$$

with inhomogeneous boundary data, provided in the weighted Orlicz setting. Result will be based on papers obtained together with Raj Narayan Dhara and Miroslav Krbec.

Martin Kalousek (Polish Academy of Sciences)
Homogenization of nonlinear elliptic systems in nonreflexive Musielak-Orlicz spaces

The talk is devoted to the homogenization process for families of strongly nonlinear elliptic systems with the homogeneous Dirichlet boundary conditions. The growth and the coercivity of the elliptic operator is assumed to be indicated by a general inhomogeneous anisotropic N–function M, which may be possibly also dependent on the spatial variable, i.e., the homogenization process will change the characteristic function spaces at each step. Such a problem is well known and there exist many positive results for the L^p-setting with restrictions on constant exponent or variable exponent that is assumed to be additionally log-Hölder continuous. These situations correspond to a very particular case of N–functions satisfying Δ_2 and ∇_2–conditions. We shall show that for M satisfying a condition of log-Hölder type one can avoid all difficulties and provide a rather general theory without any assumption on the validity of Δ_2 or ∇_2 conditions.

Filip Klawe (University of Heidelberg)
Thermo-visco-elasticity for models with growth conditions in Orlicz spaces

Our research is directed to a quasi-static evolution of the thermo-visco-elastic model. We assume that the material is subject to two kinds of mechanical deformations: elastic and inelastic. Moreover, appearance of inelastic deformation cause also the changes of materials temperature. Since constitutive function on evolution of visco-elastic deformation depends on temperature, the visco-elastic properties of material also depend on temperature. We consider the thermodynamically complete model related to hardening rule with growth condition in generalized Orlicz spaces. We provide the proof of existence of solutions for such class of models.

Josef Málek (Charles University)

On analysis of several nonstandard growth problems in fluid and solid mechanics

In the talk, we present several nonstandard growth problems in fluid and solid mechanics that come out naturally from *implicit constitutive equations*, paying a particular attention to the results concerning their analysis. We will concentrate on the problems in the following areas:

- implicitly constituted incompressible fluids [1, 2, 3],
- incompressible fluids flowing through porous (rigid) media with pressure dependent porosity and viscosity [4, 5, 6],
- incompressible fluids of power-law type with concentration/pressure dependent power-law index [7, 8, 9],
- compressible fluids with bounded divergence [10],
- solids with bounded linearized strain [11, 12, 13, 14].

[1] M. Bulíček and P. Gwiazda and J. Málek and A. Świerczewska-Gwiazda. *On unsteady flows of implicitly constituted incompressible fluids.* SIAM J. Math. Anal. **44** (2012) 2756–2801.

[2] M. Bulíček and P. Gwiazda and J. Málek and K. R. Rajagopal and A. Świerczewska-Gwiazda. *On flows of fluids described by an implicit constitutive equation characterized by a maximal monotone graph.* Mathematical Aspects of Fluid Mechanics (Eds. J. C. Robinson, J. L. Rodrigo and W. Sadowski), London Mathematical Society Lecture Note Series (No. 402) (2012) Cambridge University Press, 23–51.

[3] M. Bulíček and J. Málek and E. Süli. *Existence of Global Weak Solutions to Implicitly Constituted Kinetic Models of Incompressible Homogeneous Dilute Polymers.* Communications in Partial Differential Equations **38** (2013) 882–924.

[4] M. Bulíček and J. Málek and J. Žabenský. *A generalization of the Darcy-Forchheimer equation involving an implicit, pressure-dependent relation between the drag force and the velocity.* Journal of Mathematical Analysis and Applications, **424** (2015) 785–801.

[5] M. Bulíček and J. Málek and J. Žabenský. *On generalized Stokes' and Brinkman's equations with pressure- and shear-dependent viscosity and drag coefficient.* Nonlinear Analysis - Real World Applications August **26** (2015) 109–132.

[6] M. Bulíček and J. Žabenský. *Large data existence theory for unsteady flows of fluid with a pressure- and shear-dependent viscosity* Nonlinear Analysis - Theory,methods & Applications **127** (2015) 94–127.

[7] M. Bulíček and P. Pustějovská *On existence analysis of steady flows of generalized Newtonian fluids with concentration dependent power-law index*, J. Math. Anal. Appl., 402 (2013) 157–166.

[8] M. Bulíček and P. Pustějovská *Existence analysis for a model describing flow of an incompressible chemically reacting non-Newtonian fluid*, SIAM J. Math. Anal., 46 (2014) 3223–3240.

[9] J. Málek and K.R. Rajagopal and J. Žabenský. *On power-law fluids with the power-law index proportional to the pressure.* Appl. Math. Lett. **62** (2016) 118–123.

[10] E. Feireisl and X. Liao and J. Málek. *Global weak solutions to a class of non-Newtonian compressible fluids.* (2015) accepted for publication in Mathematical Methods in the Applied Sciences

[11] M. Bulíček and J. Málek and K. R. Rajagopal and J. Walton. *Existence of solutions for the anti-plane stress for a new class of "strain-limiting" elastic bodies.* Communications in PDEs and Calculus of Variations **54** (2015) 2115-2147

[12] M. Bulíček and J. Málek and E. Süli. *Analysis and approximation of a strain-limiting nonlinear elastic model.* Mathematics and Mechanics of Solids, **20**(1) (2014) 92–118

[13] M. Bulíček and J. Málek and E. Süli and K. R. Rajagopal. *On elastic solids with limiting small strain: modelling and analysis.* EMS Surveys in Mathematical Sciences **1** (2014) 283-332

[14] L. Beck and M. Bulíček and J. Málek and E. Süli. *On the existence of integrable solutions to variational problens and nonlinear elliptic systems with linear growth.* Preprint of MORE project. (2015)

Erika Maringová (Charles University)
Globally Lipschitz minimizers for variational problems with linear growth

The classical example of a variational problem with linear growth is the minimal surface problem, which for smooth data possesses a regular solution if the domain is convex. On the other hand, for non-convex domains there always exist data for which the solution does exist only in the space BV (the trace is not attained). In the work we sharply identify the class of functionals (the minimal surface problem being prototypic example) for which we always have regular (up to the boundary) solution in any dimension for arbitrary regular domain. This is a joint work with L. Beck and M. Bulíček.

Katarzyna Mazowiecka (University of Freiburg/University of Warsaw)
Fractional div-curl quantities and applications to nonlocal geometric equations

We investigate a fractional notion of gradient and divergence operator. We generalize the div-curl estimate by Coifman-Lions-Meyer-Semmes to fractional div-curl quantities. We demonstrate how these quantities appear naturally in nonlocal geometric equations, which can be used to obtain a regularity theory for fractional harmonic maps and critical systems with nonlocal antisymmetric potential. Joint work with Armin Schikorra.

Sebastian Schwarzacher (Charles University)
Higher integrability for the porous medium equation

The talk is about self improving properties for the gradient of solutions to degenerate parabolic equations of porous medium-type. First, that gradients of solutions satisfy a reverse Holder inequality on suitable intrinsic cylinders. And second, the local higher integrability property of gradients of solutions which can be deduced from the reverse Hoelder inequality. This can be achieved by modifying the by-now classical Gehring lemma to a setting with an intrinsic Calderon-Zygmund covering.

Iwona Skrzypczak (Polish Academy of Sciences)
Renormalized solutions to elliptic equation in Musielak-Orlicz space

We prove existence of renormalized solutions to general nonlinear elliptic equation in Musielak-Orlicz space avoiding growth restrictions. Namely, we consider

$$-\mathrm{div} A(x, \nabla u) = f \in L^1(\Omega),$$

on a Lipschitz bounded domain in $\mathbb{R}^n$. The growth of the monotone vector field A is controlled by a generalized nonhomogeneous and anisotropic N-function M. The approach does not require any particular type of growth condition of M or its conjugate M^* (neither Δ_2, nor ∇_2). The condition we impose is log-Hölder continuity of M, which results in good approximation properties of the space. The proof of the main results uses truncation ideas, the Young measures methods and monotonicity arguments.

This is a joint work with Piotr Gwiazda and Anna Zatorska–Goldstein.

[1] P. Gwiazda, I. Skrzypczak, A. Zatorska-Goldstein, *Existence of renormalized solutions to elliptic equation in Musielak-Orlicz space*, arXiv:1701.08970.

Agnieszka Świerczewska-Gwiazda (University of Warsaw)
Non-Newtonian fluids in the setting of Orlicz spaces

During the talk we will discuss several issues of non-Newtonian fluids with a property that the growth of the highest order nonlinear term is prescribed by means of an $N-$function. In a consequence we will use the framework of Orlicz spaces or Musielak-Orlicz spaces.

We consider flows of incompressible fluids with rheology given by an implicit constitutive equation relating the Cauchy stress and the symmetric part of the velocity gradient in such a way that it leads to a maximal monotone (possibly multivalued) graph. Such a framework includes standard Navier-Stokes and power-law fluids, Bingham fluids, Herschel- Bulkley fluids, and shear-rate dependent fluids with discontinuous viscosities as special cases.

Using tools such as the Young measures, properties of spatially dependent maximal monotone operators and Lipschitz approximations of Sobolev functions, we are able to extend the results concerning large data existence of weak solutions to those values of the power-law index that are of importance from the point of view of engineering and physical applications.

In the second part of the talk we discuss the issue how the notion of modular convergence is essentially used to relax assumptions of an $N-$function in the problems of fluid dynamics. The talked is based on the following papers:

[1] M. Bulíček, P. Gwiazda, J. Málek, A. Świerczewska-Gwiazda, *On unsteady flows of implicitly constituted incompressible fluids*. SIAM J. Math. Anal. 44 (2012), no. 4, 2756–2801.

[2] P. Gwiazda, A. Świerczewska-Gwiazda, A. Wróblewska, *Generalized Stokes system in Orlicz spaces.* Discrete Contin. Dyn. Syst. 32 (2012), no. 6, 2125–2146.

[3] P. Gwiazda, A. Świerczewska-Gwiazda, *On non-Newtonian fluids with a property of rapid thickening under different stimulus.* Math. Models Methods Appl. Sci. 18 (2008), no. 7, 1073–1092.

[4] P. Gwiazda, A. Świerczewska-Gwiazda, *On steady non-Newtonian fluids with growth conditions in generalized Orlicz spaces.* Topol. Methods Nonlinear Anal. 32 (2008), no. 1, 103–113.

[5] A. Świerczewska-Gwiazda, *Anisotropic parabolic problems with slowly or rapidly growing terms.* Colloq. Math. 134 (2014), no. 1, 113–130.

[6] A. Świerczewska-Gwiazda, *Nonlinear parabolic problems in Musielak-Orlicz spaces.* Nonlinear Anal. 98 (2014), 48–65.

Olli Toivanen (University of Umeå)
Regularity on generalized Orlicz spaces

I will review recent results by Harjulehto, Hästö, Cruz-Uribe, myself and others in the regularity theory of generalized Orlicz spaces. These spaces generalize widely disparate cases, including Orlicz, L^p, and $L^{p(\cdot)}$. Recent work has began to suggest generalizations of their various assumptions for regularity.

Aneta Wróblewska-Kamińska (Imperial College London)
Non-Newtonian fluids and abstract problems:
applications of Orlicz spaces in the theory of nonlinear PDE

We are interested in the existence of solutions to strongly nonlinear partial differential equations. We concentrate mainly on problems which come from dynamics of non-Newtonian fluids of a nonstandard rheology, more general then of power-law type, and also on some abstract theory of elliptic and parabolic equations. In considered problems the nonlinear highest order term (stress tensor) is monotone and its behaviour — coercivity/growth condition — is given with help of some general convex function. In our research we would like to cover both cases: sub- and super-linear growth of nonlinearity (shear thickening and shear tinning fluids) as well its anisotropic and non-homogenous behaviour. Such a formulation requires a general framework for the function space setting, therefore we work with non-reflexive and non-separable anisotropic Orlicz and Musielak-Orlicz spaces. Within the presentation we would like to emphasise problems we have met during our studies, their reasons and methods which allow us to achieve existence results.

Ahmed Youssfi (Sidi Mohamed Ben Abdellah University)
Nonlinear parabolic equations with variable nonlinearities

We study the solvability of the initial-boundary value problem for second-order parabolic equations governed by nonlinear operators in divergence form with nonstandard growth conditions and L^2-source terms. In the model case, these equations include the p-Laplacian with a variable exponent p. In suitable functional frameworks, we prove that if the variable exponent p is bounded away from both 1 and $+\infty$ and is log-Hölder continuous, then the problem has a weak solution which satisfies the energy equality. The talk follows papers [1,2].

[1] A. Youssfi, E. Azroul and B. Lahmi, *Nonlinear parabolic equations with nonstandard growth*, Appl. Anal., 2016, 95 (12), 2766-2778.

[2] E. Azroul, B. Lahmi and A. Youssfi, *Strongly nonlinear variational parabolic equations with $p(x)$-growth*. Acta Mathematica Scientia 2016, 36B (5):1383-1404.