Book of Abstracts

# Workshop
# Current Topics in Kinetic Theory

Institute of Mathematics of Polish Academy of Sciences in Warsaw
Banach Center, Warsaw, 27-29 March 2017

Organizing committee: José A. Carillo, Piotr Gwiazda, Benoît Perthame and Agnieszka Świerczewska-Gwiazda
Local Organizers: Tomasz Dębiec, Kamila Łyczek

Banach Center
INSTITUTE OF MATHEMATICS PAS

Warsaw Center
of Mathematics
and Computer Science

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# Schedule

| | Monday, March 27<br>Room 403 | Tuesday, March 28<br>Room 321 | Wednesday, March 29<br>Room 321 |
|---|---|---|---|
| 8:30–9:00 | Registration, opening and coffee | Coffee start | Coffee start |
| 9:00–9:45 | Eitan Tadmor<br>*Hydrodynamic flocking: pressure-less equations and alignment-based commutator structure* | Endre Süli<br>*Existence of global weak solutions to the kinetic Hookean dumbbell model for incompressible dilute polymeric fluids* | Andrea Tosin<br>*Kinetic description of collision avoidance in pedestrian crowds by sidestepping* |
| 9:50–10:35 | Vincent Calvez<br>*Traveling waves of chemotactic bacteria* | Laurent Desvillettes<br>*New results of smoothness for spatially inhomogeneous coagulation-fragmentation equations* | Rinaldo Colombo<br>*Structured Population Models: Analysis and Control* |
| 10:40–11:10 | Coffee break | Coffee break | Coffee break |
| 11:10–11:55 | Christian Schmeiser<br>*Hypocoercivity for Kinetic Problems: Fourier Mode Analysis, Factorization, Nonlinearity* | Marie Doumic<br>*Asymptotic behaviour of critical cases in the fragmentation and growth-fragmentation equation - cyclic and non steady behaviours* | Paola Goatin<br>*Non-local macroscopic models of traffic-flow* |
| 12:00–14:00 | Lunch break | Lunch break | |
| 14:00–14:45 | Pierre-Emannuel Jabin<br>*A new statistical approach to the propagation of chaos for large systems of interacting particles* | Konstantina Trivisa<br>*Analysis on models of polymeric fluids* | |
| 14:50–15:35 | Miroslav Buliček<br>*On unified theory for scalar conservation laws with fluxes and sources discontinuous with respect to the unknown* | Christina Surulescu<br>*Some model classes for tumor invasion in tissue* | |
| 15:40–16:10 | Coffee break | Coffee break | |
| 16:10–16:55 | Poster session | Grzegorz Karch<br>*Eternal and infinite energy solutions of homogeneous Boltzmann equitation* | |
| 18:30 | | Dinner | |

## On unified theory for scalar conservation laws with fluxes and sources discontinuous with respect to the unknown

**Miroslav Buliček**

Charles University

### Abstract

We deal with the Cauchy problem for multi-dimensional scalar conservation laws, where the fluxes and the source terms can be discontinuous functions of the unknown and show that a proper form of the kinetic formulation is equivalent to re-parametrization of the flux and the source functions, which was previously used for Kružkov entropy solutions. Within this approach we obtain a fairly complete picture: existence of entropy measure valued solutions, entropy weak solutions and their equivalence to the kinetic solution. The results of existence and uniqueness follow under the assumption of Hölder continuity at zero of the flux. The source term, what is another novelty for the studies on problems with discontinuous flux, is only assumed to be one-side Lipschitz, not necessarily monotone function.

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## Traveling waves of chemotactic bacteria

**Vincent Calvez**

ENS Lyon

### Abstract

I will present some recent analytical and numerical results about the existence of traveling wave solutions for a coupled kinetic-parabolic system describing concentration waves of bacteria in a micro-channel. The parabolic-parabolic problem obtained in the diffusive limit admits unique traveling wave solutions without any restriction on the parameters. This is in opposition to the kinetic-parabolic system for which solutions may be not unique, or may not exist for some extreme range of parameters.

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## Structured Population Models: Analysis and Control

**Rinaldo Colombo**

University of Brescia

### Abstract

Various structured population models lead to initial-boundary value problems for balance laws, often equipped with non-local boundary conditions. The present talks overviews recent results on the well posedness of such models. Different analytic environments can be considered: measure spaces and sets of BV functions, for instance. In the latter case, examples are shown where realistic modeling requires renewal equations to be set on graphs. Then, different control problems are stated and, where possible, an effective strategy to tackle them is proposed and pursued. Information obtained through analytic techniques allow to significantly shorten the numerical search for optimal controls.

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## New results of smoothness for spatially inhomogeneous coagulation-fragmentation equations

**Laurent Desvillettes**

ENS Lyon

### Abstract

Coagulation fragmentation equations were introduced by Smoluchowsky at the beginning of the 20th century and are widely used in (for example) the modeling of polymers. They consist (in the spatially inhomogenous setting) of an infinite system of reaction-diffusion equations involving series in the r.h.s (reaction) part of the system. Duality methods were imported some years ago from the field of standard reaction-diffusion equations in order to study the possibility of gelation (from the point of view of PDEs: blow up of solutions). We present in this talk results obtained in a collaboration with M. Breden and K. Fellner in which a refined version of the duality lemmas enables to show that in typical situations when gelation does not occur, smoothness is propagated for the moments of the solutions.

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## Asymptotic behaviour of critical cases in the fragmentation and growth-fragmentation equation - cyclic and non steady behaviours

**Marie Doumic**

INRIA

### Abstract

The long-time asymptotics of the fragmentation and growth-fragmentation equations have been studied by many authors, mainly proving convergence toward a steady behaviour (possibly with exponential growth) under balance assumptions on the coefficients. Exponential speed of convergence has also been established under more restrictive assumptions. We focus here on two limit cases where no such behaviour occurs. The first case is such that the balance assumptions between growth and division are not satisfied. We show that a specific dynamics emerge, where the initial condition continues to play a major role in the asymptotic profile. In the second case, emblematic of bacterial division cycle, a lack of dissipativity in the operator leads to a convergence toward a periodic behaviour. Common work with E. Bernard, M. Escobedo, P. Gabriel.

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## Non-local macroscopic models of traffic-flow

**Paola Goatin**

INRIA

### Abstract

Non-local interactions can be described through macroscopic models based on integro-differential equations. Systems of the form

$$\partial_t u + \operatorname{div}_x F(t, x, u, W) = 0, \qquad t > 0,\ x \in \mathbb{R}^d,\ d \geq 1$$

where $u = u(t, x) \in \mathbb{R}^N$, $N \geq 1$ is the vector of the conserved quantities and the variable $W = W(t, x, u)$ depends on an integral evaluation of $u$, arise in a variety of physical applications. Space-integral terms are considered for example in models for granular flows, sedimentation, supply chains, conveyor belts and biological applications like structured population dynamics. In particular, equations with non-local flux have been recently introduced in traffic flow modelling to account for the reaction of drivers or pedestrians to the surrounding density of other individuals. While pedestrians are likely to react to the presence of people all around them, drivers mainly adapt their velocity to the downstream traffic, assigning a greater importance to closer vehicles. In particular, and in contrast to classical (without integral terms) macroscopic equations, these models are able to display finite acceleration of vehicles through Lipschitz bounds on the mean velocity and lane formation in crossing pedestrian flows. We will also present recent results on micro-macro limits of empirical measures converging to measure-valued solutions of the corresponding macroscopic evolution equation.

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## A new statistical approach to the propagation of chaos for large systems of interacting particles

**Pierre-Emannuel Jabin**

University of Maryland

### Abstract

We introduce a new approach for large systems of particles that is based on the Gibbs entropy at the scale of the Liouville equation. This method leads to explicit and quantitative estimates on the propagation of chaos together with an optimal rate of convergence to the Mean Field limit. In addition as it does not use any well posedness on the characteristics, we only require very weak regularity on the interaction kernel: Bounded without any need for differentiability. The estimates themselves rely on modified law of large numbers proved by careful combinatorics methods. This is a joint work with Z. Wang.

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## Eternal and infinite energy solutions of homogeneous Boltzmann equitation

**Grzegorz Karch**

University of Wroclaw

### Abstract

I will announce our recent results on the existence of eternal solutions to the homogeneous Boltzmann equation for Maxwellian molecules. Such solutions are obtained in a space of probability measures of infinite energy (*i.e.* infinite second moment). They describe the large time behavior of other infinite energy solutions and appear as well as intermediate time asymptotic states of finite, but arbitrary high, energy solutions.

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## Hypocoercivity for Kinetic Problems: Fourier Mode Analysis, Factorization, Nonlinearity

**Christian Schmeiser**

University of Vienna

### Abstract

Extensions of the hypocoercivity approach developed together with J. Dolbeault and C. Mouhot will be presented. A mode-by-mode analysis provides decay results for problems without confinement. A factorization method permits stronger results in terms of assumptions on initial data. Finally the potential of the approach to deal with nonlinear problems is demonstrated on a reaction-kinetic system. (Joint work with E. Bouin, J. Dolbeault, S. Mischler, C. Mouhot, L. Neumann)

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## Existence of global weak solutions to the kinetic Hookean dumbbell model for incompressible dilute polymeric fluids

**Endre Süli**

University of Oxford

### Abstract

We consider a longstanding open question, concerned with the existence of global-in-time weak solutions to the Hookean dumbbell model, which arises as a microscopic-macroscopic bead-spring model from the kinetic theory of dilute solutions of polymeric liquids with noninteracting polymer chains. This model involves the unsteady incompressible Navier-Stokes equations in a bounded domain in two or three space dimensions for the velocity and the pressure of the fluid, with an elastic extra-stress tensor appearing on the right-hand side in the momentum equation. The extra-stress tensor stems from the random movement of the polymer chains and is defined by the Kramers expression through the associated probability density function that satisfies a Fokker-Planck-type parabolic equation, a crucial feature of which is the presence of a center-of-mass diffusion term. We show the existence of large-data global weak solutions in the case of two space dimensions. Indirectly, our proof also rigorously demonstrates that the Oldroyd-B model is a macroscopic closure of the Hookean dumbbell model in two space dimensions. Finally, we show the existence of large-data global weak subsolutions to the Hookean dumbbell model in the case of three space dimensions. The talk is based on recent joint work with John W. Barrett (Imperial College, London).
The preprint of our paper is available from https://arxiv.org/abs/1702.06502

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## Some model classes for tumor invasion in tissue

**Christina Surulescu**

TU Kaiserslautern

### Abstract

We present several model classes for cancer invasion in tissue networks, the settings involving couplings between different types of equations and accounting for one or several scales for the modeled processes. Thus, the models include: kinetic transport equations coupled with ODEs (micro-meso model) leading via parabolic scaling to a reaction-diffusion-haptotaxis equation; pure macroscopic models with haptotaxis and diverse kinds of degenerate diffusion; and a state-structured population model with cell-cell and cell-tissue adhesions. Joint work with C. Engwer (WWU Münster), A. Hunt (TU Kaiserslautern), C. Stinner (TU Darmstadt), M. Winkler (Univ. Paderborn), and A. Zhigun (TU Kaiserslautern)

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## Hydrodynamic flocking: pressure-less equations and alignment-based commutator structure

**Eitan Tadmor**

University of Maryland/ETH Zürich

### Abstract

We discuss the question of global regularity for a general class of Eulerian dynamics driven by a forcing with a commutator structure. The study of such systems is motivated by the hydrodynamic description of agent-based models for flocking driven by alignment.

For commutators involving bounded kernels, existence of strong solutions follows for initial data which are sub-critical, namely – the initial divergence is "not too negative" and the initial spectral gap is "not too large". Singular kernels, corresponding to fractional Laplacian behave better: global regularity persists and flocking follows. Singularity helps! A similar role of the spectral gap appears in the study of two-dimensional pressure-less equations. Here, we develop a new BV framework to prove the existence of weak dual solutions for the 2D pressure-less Euler equations as vanishing viscosity limits.

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## Kinetic description of collision avoidance in pedestrian crowds by sidestepping

**Andrea Tosin**

Politecnico di Torino

### Abstract

We consider a collision avoidance mechanism between pedestrians based on sidestepping: when a pedestrian estimates that s/he is going to collide with a neighbouring walker s/he deviates leftwards or rightwards by a certain angle $\alpha_c$. S/he evaluates dynamically the probability to collide by assessing the time to collision, i.e. an extrapolation of the time needed to get closer to the other pedestrian than a certain given distance. As a complementary action, s/he also tries to walk in a desired direction identified by another angle $\alpha_d$. These minimal microscopic rules lead to complex emergent macroscopic phenomena, such as the alignment of the velocity in unidirectional flows and the lane or stripe formation in bidirectional flows. We discuss the collision avoidance mechanism at the microscopic scale, then we study the corresponding Boltzmann-type kinetic description and its mean-field approximation in the quasi-invariant direction limit. In the spatially homogeneous case we prove directional alignment under specific conditions on the sidestepping rules for both the collisional and the mean-field model. In the spatially inhomogeneous case we illustrate numerically the rich dynamics that the proposed model is able to reproduce. This is a joint work with Adriano Festa (Rouen, France) and Marie-Therese Wolfram (Warwick, UK).

Reference: A. Festa, A. Tosin, M.-T. Wolfram. Kinetic description of collision avoidance in pedestrian crowds by sidestepping, arXiv:1610.05056, 2016.

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# Analysis on models of polymeric fluids

**Konstantina Trivisa**

University of Maryland

### Abstract

The Doi model for the suspensions of rod-like molecules in a dilute regime describes the interaction between the orientation of rod-like polymer molecules on the micro-scopic scale and the macroscopic properties of the uid in which these molecules are contained (cf. Doi and Edwards (1986)). The orientation distribution of the rods on the microscopic level is described by a Fokker-Planck-type equation on the sphere, while the fluid flow is given by the Navier-Stokes equations, which are now enhanced by an additional macroscopic stress reecting the orientation of the rods on the molecular level. This talk will focus on three distinct but related models for polymeric fluids and presents results on the well-posedness of solutions.